લિબર્ટી પેપરસેટ

ધોરણ 12 : ગણિત

Full Solution

સમય : 3 કલાક

અસાઈનમેન્ટ પ્રશ્નપત્ર 5

PART A

1. (A) **2.** (C) **3.** (A) **4.** (C) **5.** (B) **6.** (A) **7.** (D) **8.** (C) **9.** (B) **10.** (D) **11.** (D) **12.** (A) **13.** (C) **14.** (C) **15.** (A) **16.** (A) **17.** (B) **18.** (D) **19.** (A) **20.** (B) **21.** (B) **22.** (C) **23.** (C) **24.** (A) **25.** (B) **26.** (A) **27.** (B) **28.** (C) **29.** (C) **30.** (B) **31.** (C) **32.** (B) **33.** (D) **34.** (B) **35.** (D) **36.** (A) **37.** (C) **38.** (C) **39.** (A) **40.** (C) **41.** (D) **42.** (B) **43.** (A) **44.** (D) **45.** (C) **46.** (C) **47.** (A) **48.** (B) **49.** (C) **50.** (D)

PART B

વિભાગ-A

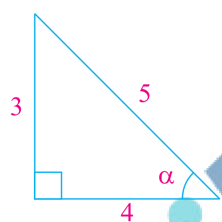
1.

રીત 1 :

ધારો કે $sin^{-1}\frac{3}{5} = \alpha$

$\therefore \frac{3}{5} = sin\ \alpha$

$\therefore tan\ \alpha = \frac{3}{4}$



તે જ પ્રમાણે,

ધારો કે $cos^{-1}\frac{3}{2} = \beta$

$\therefore \frac{3}{2} = cot\ \beta$

$\therefore tan\ \beta = \frac{2}{3}$

$\therefore tan\left(sin^{-1}\frac{3}{5} + cot^{-1}\frac{3}{2}\right)$

$= tan\ (\alpha + \beta) = \frac{tan\ \alpha + tan\ \beta}{1 - tan\ \alpha\ tan\ \beta}$

$= \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4}\cdot\frac{2}{3}}$

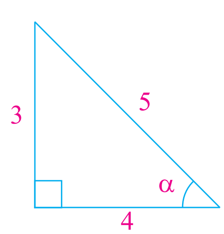
$= \frac{9+8}{12-6}$

$= \frac{17}{6}$

રીત 2 :

$tan\left(sin^{-1}\frac{3}{5} + cot^{-1}\frac{3}{2}\right)$

અહીં, $sin^{-1}\frac{3}{5} = tan^{-1}\frac{3}{4}$



$= tan\left(tan^{-1}\frac{3}{4} + tan^{-1}\frac{2}{3}\right)$

$= tan\left[tan^{-1}\left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \times \frac{2}{3}}\right)\right] \quad \left(\frac{3}{4}\cdot\frac{2}{3} < 1\right)$

$= tan\left[tan^{-1}\left(\frac{9+8}{12-6}\right)\right]$

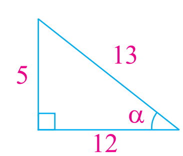
$= tan\left[tan^{-1}\left(\frac{17}{6}\right)\right]$

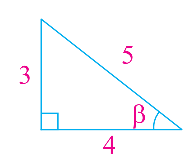
$= \frac{17}{6}$

2.

ડા.બા. $= cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5}$

$cos^{-1}\frac{12}{13} = \alpha \quad , \quad sin^{-1}\frac{3}{5} = \beta$





$\therefore cos\ \alpha = \frac{12}{13}, sin\ \alpha = \frac{5}{12} \quad \Big| \quad sin\ \beta = \frac{3}{5}, cos\ \beta = \frac{4}{5}$

$sin(\alpha + \beta) = sin\ \alpha\ cos\ \beta + cos\ \alpha\ sin\ \beta$

$= \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{3}{5}\right)$

$= \frac{20}{65} + \frac{36}{65}$

$= \frac{56}{65}$

$\therefore \quad \alpha + \beta = sin^{-1}\frac{56}{65}$

$\therefore \quad cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5} = sin^{-1}\frac{56}{65}$

3.

➪ બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$\therefore 2x + x \cdot \frac{dy}{dx} + y + 2y \cdot \frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx}(x + 2y) = -2x - y$

$\therefore \frac{dy}{dx} = \frac{-2x - y}{x + 2y}$

4.

➪ $I = \int \frac{sec^2 x}{\sqrt{tan^2 x + 4}} dx$

અહીં, $tan\ x = t$ આદેશ લેતાં,

$\therefore sec^2 x\ dx = dt$

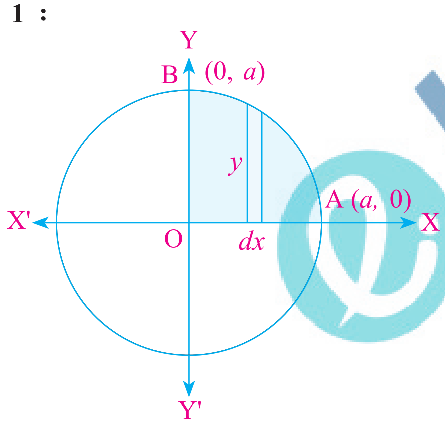
$\therefore I = \int \frac{dt}{\sqrt{t^2 + (2)^2}}$

$= log\ |t + \sqrt{t^2 + 4}\ | + c$

$\therefore I = log\ |tan\ x + \sqrt{tan^2 x + 4}\ | + c$

5.

➪ **રીત 1 :**



આકૃતિમાં દર્શાવ્યા પ્રમાણે આપેલ વર્તુળ દ્વારા આવૃત્ત પ્રદેશનું ક્ષેત્રફળ = 4 × (આપેલ વક્ર રેખા $x = 0$, $x = a$ અને X-અક્ષ દ્વારા આવૃત્ત પ્રદેશ AOBAનું ક્ષેત્રફળ). (વર્તુળ એ X-અક્ષ અને Y-અક્ષ પ્રત્યે સંમિત છે.)

માંગેલ ક્ષેત્રફળ $= 4\int_0^a y\ dx$ (શિરોલંબ પટ્ટીઓ લેતાં)

$= 4\int_0^a \sqrt{a^2 - x^2}\ dx$

હવે, $x^2 + y^2 = a^2$ પરથી $y = \pm\sqrt{a^2 - x^2}\ dx$ મળશે. અહીં પ્રદેશ AOBA પ્રથમ ચરણમાં આવેલો છે. તેથી $y = \sqrt{a^2 - x^2}$ લઈશું. આપણને વર્તુળ દ્વારા આવૃત્ત સમગ્ર પ્રદેશનું ક્ષેત્રફળ સંકલન કરતાં મળશે.

માંગેલ ક્ષેત્રફળ $= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}sin^{-1}\frac{x}{a}\right]_0^a$

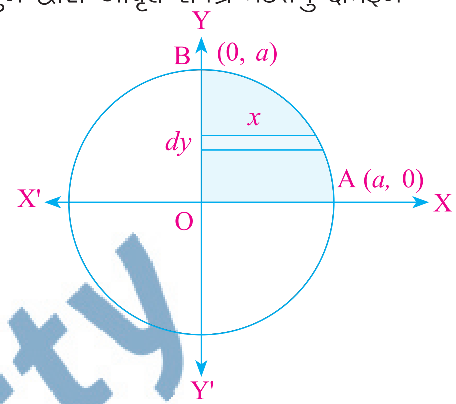
$= 4\left[\left(\frac{a}{2} \times 0 + \frac{a^2}{2}sin^{-1}1\right) - 0\right]$

$= 4\left(\frac{a^2}{2}\right)\left(\frac{\pi}{2}\right)$

$= \pi a^2$ ચો. એકમ

➪ **રીત 2 :**

આકૃતિમાં દર્શાવ્યા પ્રમાણે સમક્ષિતિજ પટ્ટીઓ લેતાં, આપેલ વર્તુળ દ્વારા આવૃત્ત સમગ્ર પ્રદેશનું ક્ષેત્રફળ



$= 4\int_0^a x\ dy$

$= 4\int_0^a \sqrt{a^2 - y^2}\ dy$

$= 4\left[\frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2}sin^{-1}\frac{y}{a}\right]_0^a$

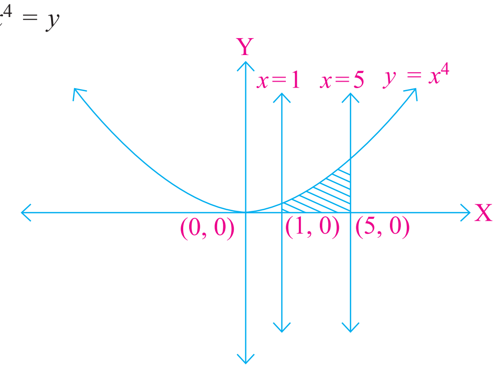
$= 4\left[\left(\frac{a}{2} \times 0 + \frac{a^2}{2}sin^{-1}1\right) - 0\right]$

$= 4\ \frac{a^2}{2}\ \frac{\pi}{2}$

$= \pi a^2$ ચો. એકમ

6.

➪ $x^4 = y$



આવૃત્ત પ્રદેશનું ક્ષેત્રફળ,

A = |I|

$\therefore\ I = \int_1^5 y\, dx$

$\therefore\ I = \int_1^5 x^4\, dx$

$\therefore\ I = \left[\frac{x^5}{5}\right]_1^5$

$\therefore\ I = \frac{1}{5}\ ((5)^5 - 1)$

$\therefore\ I = \frac{1}{5}\ (3125 - 1)$

$\therefore\ I = 624.8$

હવે, A = |I|

= |624.8|

∴ A = 624.8 ચોરસ એકમ

7.

$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

$\therefore\ \frac{dy}{1+y^2} = (1 + x^2)\ dx$

→ બંને બાજુ સંકલન કરતાં,

$\therefore\ \int \frac{dy}{y^2+1} = \int (x^2 + 1)\ dx$

$\therefore\ tan^{-1}y = \left(\frac{x^3}{3} + x\right) + c$

$\therefore\ tan^{-1}y = \frac{x^3}{3} + x + c;$

જે આપેલા વિકલ સમીકરણનો વ્યાપક ઉકેલ છે.

8.

આપેલ સદિશ $\vec{a}$ ની દિશામાં એકમ સદિશ

$\hat{a} = \frac{1}{|\vec{a}|}\ \vec{a}$

$= \frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j})$

$= \frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$

તેથી, $\vec{a}$ ની દિશામાં 7 માનવાળો સદિશ

$7\hat{a} = 7\left(\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}\right)$

$= \frac{7}{\sqrt{5}}\hat{i} - \frac{14}{\sqrt{5}}\hat{j}$

9.

$L : \frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$

$\therefore\ \frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2}$

$L : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + \frac{2p}{7}\hat{j} + 2\hat{k})$

$\therefore\ \vec{b_1} = -3\hat{i} + \frac{2p}{7}\hat{j} + 2\hat{k}$

હવે, $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

$\therefore\ \frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$

$M : \vec{r} = (\hat{i} + 5\hat{j} + 6\hat{k}) + \mu\left(\frac{-3p}{7}\hat{i} + \hat{j} - 5\hat{k}\right)$

$\therefore\ \vec{b_2} = \frac{-3p}{7} + \hat{j} - 5\hat{k}$

→ L અને M પરસ્પર લંબ હોવાથી;

$\vec{b_1} \cdot \vec{b_2} = 0$

$\therefore\ \left(-3\hat{i} + \frac{2p}{7}\hat{j} + 2\hat{k}\right)\cdot\left(-\frac{3p}{7}\hat{i} + \hat{j} - 5\hat{k}\right) = 0$

$\therefore\ \frac{9p}{7} + \frac{2p}{7} - 10 = 0$

$\therefore\ \frac{11p}{7} = 10$

$\therefore\ p = \frac{70}{11}$

10.

રેખા પરનું બિંદુ $A(\vec{a}) = -2\hat{i} + 4\hat{j} - 5\hat{k}$

રેખા $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ ને સમાંતર છે.

રેખાની દિશા $\vec{b} = 3\hat{i} + 5\hat{j} + 6\hat{k}$

∴ સમાંતર રેખાનું સમીકરણ

$\frac{x-x_1}{l_1} = \frac{y-y_1}{l_2} = \frac{z-z_1}{l_3}$

$\frac{x-(-2)}{3} = \frac{y-4}{5} = \frac{z-(-5)}{6}$

$\therefore\ \frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

જે માંગેલ રેખાનું કાર્તેઝિય સમીકરણ છે.

11.

ખોખામાં 10 કાળા રંગના દડા અને

8 લાલ રંગના દડા છે.

પુરવણી સહિત,

$$= 2 \times \frac{^{10}C_1}{^{18}C_1} \times \frac{^{8}C_1}{^{18}C_1} = \frac{40}{81}$$

12.

$P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$, $P(E \cap F) = \frac{1}{5}$

$\therefore\ P(E) \cdot P(F) = \frac{3}{5} \times \frac{3}{10}$

$= \frac{9}{50}$

$\neq P(E \cap F)$

$\therefore$ E અને F નિરપેક્ષ ઘટનાઓ નથી.

વિભાગ-B

13.

અહીં $f : N \to N$, $f(n) = \begin{cases} \frac{n+1}{2} & n \text{ અયુગ્મ} \\ \frac{n}{2} & n \text{ યુગ્મ,} \end{cases}$

$n_1 = 3$, $n_2 = 4$ લેતાં,

$f(n_1) = \frac{3+1}{2}$ તથા $f(n_2) = f(4)$

$= 2$ $\quad = \frac{4}{2} = 2$

અહીં $n_1 \neq n_2$ પરંતુ $f(n_1) = f(n_2)$

$\therefore$ f એ એક-એક વિધેય નથી.

પ્રદેશ $N = \{1, 2, 3, 4, 5, 6, ...\}$

$f(n) = \begin{cases} \frac{n+1}{2} & n \text{ અયુગ્મ} \\ \frac{n}{2} & n \text{ યુગ્મ,} \end{cases}$

$f(1) = \frac{1+1}{2} = 1$

$f(2) = \frac{2}{2} = 1$

$f(3) = \frac{3+1}{2} = 2$

$f(4) = \frac{4}{2} = 2$

$f(5) = \frac{5+1}{2} = 3$

$f(6) = \frac{6}{2} = 3$

$\therefore R_f = \{1, 2, 3, 4 ....\} = N$ (સહપ્રદેશ)

$\therefore$ f વ્યાપ્ત વિધેય છે.

14.

$A^2 = A \,.\, A$

$= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

હવે, $A^2 = KA - 2I$

$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3K & -2K \\ 4K & -2K \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3K-2 & -2K \\ 4K & -2K-2 \end{bmatrix}$

$\therefore$ $1 = 3K - 2$	$-2 = -2K$	$4 = 4K$	$-4 = -2K + 2$
$\therefore$ $3K = 3$	$\therefore K = 1$	$\therefore K = 1$	$-2 = -2K$
$K = 1$			$\therefore K = 1$

$\therefore$ $K = 1$

15.

આપણને $A^2 = A \cdot A \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$ મળે.

આથી, $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= O$

હવે, $A^2 - 4A + I = O$

માટે $AA - 4A = -I$

અથવા $AA(A^{-1}) - 4AA^{-1} = -IA^{-1}$

($|A| \neq 0$ હોવાથી A^{-1} વડે ઉત્તર ગુણાકાર કરતાં)

અથવા $A(AA^{-1}) - 4I = -A^{-1}$

અથવા $AI - 4I = -A^{-1}$

અથવા $A^{-1} = 4I - A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

તેથી $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.

16.

ધારો કે, $u = (sinx)^x$ તથા $v = sin^{-1}\sqrt{x}$

$\therefore y = u + v$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad ... (1)$$

હવે, $u = (sinx)^x$ ની

બંને બાજુ log લેતાં,

$log\ u = x log\ sinx$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log sinx + \log sinx\frac{d}{dx}x$$

$$= x \times \frac{1}{\sin x}\cos x + \log \sin x$$

$$\therefore \frac{1}{u}\frac{du}{dx} = x \cdot cotx + logsinx$$

$$\frac{du}{dx} = u[x \cdot cotx + logsinx]$$

$$\frac{du}{dx} = (sinx)^x\ [x \cdot cotx + logsinx] \quad \dots (2)$$

હવે, $v = sin^{-1}\sqrt{x}$ ની

બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}\sqrt{x}$$

$$= \frac{1}{\sqrt{1-x}}\ \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}} \quad \dots (3)$$

પરિણામ (1) માં પરિણામ (2) અને (3) ની કિંમત મૂકતાં,

$$\frac{dy}{dx} = (sinx)^x[xcotx + logsinx] + \frac{1}{2\sqrt{x-x^2}}$$

17.

$y = log(1 + x) - \frac{2x}{2+x},\ x > -1$

$$\frac{dy}{dx} = \frac{1}{1+x} - \left[\frac{(2+x)(2)-(2x)}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \left[\frac{4+2x-2x}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \frac{4}{(2+x)^2}$$

$$= \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2}$$

$$= \frac{4+4x+x^2-4-4x}{(1+x)(2+x)^2}$$

$$\frac{dy}{dx} = \frac{x^2}{(1+x)(2+x)^2}$$

હવે, $x > -1 \Rightarrow x^2 \geq 0$

$\Rightarrow (1 + x) > 0$

$\Rightarrow (2 + x)^2 > 0$

$\Rightarrow \frac{dy}{dx} \geq 0$

$\therefore x > -1$ પર f વધતું વિધેય છે.

18.

રીત 1 :

ધારો કે $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$

હવે, $\vec{d} \perp \vec{a}$

$\therefore \vec{d} \cdot \vec{a} = 0$

$\therefore (d_1\hat{i} + d_2\hat{j} + d_3\hat{k}) \cdot (\hat{i} + 4\hat{j} + 2\hat{k}) = 0$

$\therefore d_1 + 4d_2 + 2d_3 = 0$ .... (1)

હવે, $\vec{d} \perp \vec{b}$

$\therefore \vec{d} \cdot \vec{b} = 0$

$\therefore (d_1\hat{i} + d_2\hat{j} + d_3\hat{k}) \cdot (3\hat{i} - 2\hat{j} + 7\hat{k}) = 0$

$\therefore 3d_1 - 2d_2 + 7d_3 = 0$ .... (2)

હવે, $\vec{c} \cdot \vec{d} = 15$

$\therefore (2\hat{i} - \hat{j} + 4\hat{k})(d_1\hat{i} + d_2\hat{j} + d_3\hat{k}) = 15$

$\therefore 2d_1 - d_2 + 4d_3 = 15$ .... (3)

પરિણામ (1) અને (2) નો ઉકેલ કરતાં,

$$\begin{aligned} d_1 + 4d_2 + 2d_3 &= 0 \\ 6d_1 - 4d_2 + 14d_3 &= 0 \\ \hline 7d_1 + 16d_3 &= 0 \end{aligned}$$ .... (4)

પરિણામ (2) અને (3) નો ઉકેલ કરતાં,

$$\begin{aligned} 3d_1 - 2d_2 + 7d_3 &= 0 \\ \underset{-}{4d_1} \underset{+}{-} 2d_2 \underset{-}{+} \underset{-}{8d_3} &= 30 \\ \hline -d_1 - d_3 &= -30 \end{aligned}$$

$\therefore d_1 + d_3 = 30$ .... (5)

પરિણામ (4) અને (5) નો ઉકેલ કરતાં,

$$\begin{aligned} 7d_1 + 16d_3 &= 0 \\ \underset{-}{7d_1} \underset{-}{+} 7d_3 &= \underset{-}{210} \\ \hline 9d_3 &= -210 \end{aligned}$$

$\therefore d_3 = -\frac{70}{3}$

d_3 ની કિંમત પરિણામ (5) માં મૂકતાં,

$d_1 - \frac{70}{3} = 30$

$\therefore d_1 = \frac{90+70}{3}$

$\therefore d_1 = \frac{160}{3}$

d_1, d_3 ની કિંમત પરિણામ (1) માં મૂકતાં,

$\frac{160}{3} + 4d_2 - \frac{140}{3} = 0$

$\therefore 4d_2 = -\frac{20}{3}$

$\therefore d_2 = \frac{-5}{3}$

$\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$

$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k}$

➪ **રીત 2 :**

$\vec{d}$ એ $\vec{a}$ અને $\vec{b}$ બંનેને લંબ છે.

∴ $\vec{d}$ એ $\vec{a} \times \vec{b}$ ને સમાંતર થાય.

∴ $\vec{d} = p \cdot (\vec{a} \times \vec{b})$

હવે, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$

$= \hat{i} \cdot (28+4) - \hat{j}(7-6) + \hat{k}(-2-12)$

$= 32\hat{i} - \hat{j} - 14\hat{k}$

હવે, $\vec{d} = p(\vec{a} \times \vec{b})$

$= p(32\hat{i} - \hat{j} - 14\hat{k})$ .... (1)

તેમજ $\vec{c} \cdot \vec{d} = 15$

∴ $(2\hat{i} - \hat{j} + 4\hat{k}) \cdot [p(32\hat{i} - \hat{j} - 14\hat{k})] = 15$

∴ $p[(2)(32) + (-1)(-1) + 4(-14)] = 15$

∴ $9p = 15$

∴ $p = \frac{15}{9} = \frac{5}{3}$

$p = \frac{5}{3}$ પરિણામ (1)માં મૂકતાં,

$\vec{d} = \frac{5}{3}(32\hat{i} - \hat{j} - 14\hat{k})$

$= \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$

19.

➪ (1) અને (2) ને અનુક્રમે $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ અને $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ સાથે સરખાવતાં, આપણને

$\vec{a_1} = \hat{i} + \hat{j}$, $\vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

અને $\vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$ મળે.

માટે, $\vec{a_2} - \vec{a_1} = \hat{i} - \hat{k}$

અને $\vec{b_1} \times \vec{b_2} = (2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$

$= 3\hat{i} - \hat{j} + 7\hat{k}$

તેથી $|\vec{b_1} \times \vec{b_2}| = \sqrt{9+1+49}$

$= \sqrt{59}$

આથી, આપેલી રેખાઓ વચ્ચેનું લઘુતમ અંતર

$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) . (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$= \left| \frac{3 - 0 + 7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}}$ એકમ

20.

➪ $3x + 5y \le 15$

$5x + 2y \le 10$

$x \ge 0$

$y \ge 0$

હેતુલક્ષી વિધેય $Z = 5x + 3y$

$3x + 5y = 15$ ... (i)

x	0	5
y	3	0

$5x + 2y = 10$ ... (ii)

x	0	2
y	5	0

(i) અને (ii)નો ઉકેલ,

$6x + 10y = 30$

$25x + 10y = 50$

$\underline{-\quad\quad -\quad\quad -}$

$19x = 20$

∴ $x = \frac{20}{19}$

$5y = -\frac{60}{19} + 15$

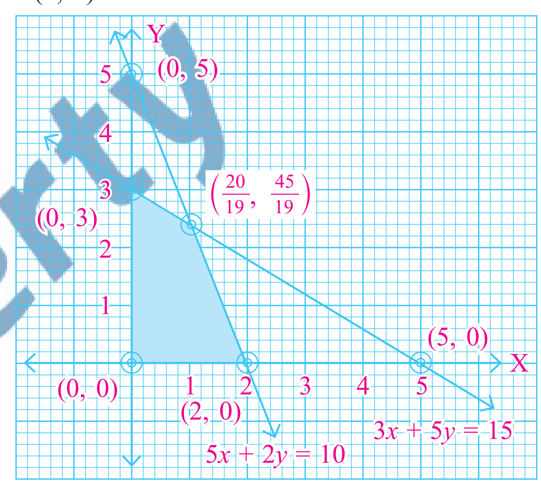
∴ $y = -\frac{12}{19} + 3$

$= \frac{-12 + 57}{19}$

∴ $y = \frac{45}{19}$

$\left(\frac{20}{19}, \frac{45}{19}\right)$

(0, 0)



આકૃતિમાં આપેલ અસમતાઓનો આલેખ દર્શાવ્યો છે જે સિમિત છે. શક્ય ઉકેલપ્રદેશનાં શિરોબિંદુઓ (0, 0), (2, 0) અને $\left(\frac{20}{19}, \frac{45}{19}\right)$ મળે.

શક્ય ઉકેલ પ્રદેશના શિરોબિંદુ	$Z = 5x + 3y$
(0, 3)	9
(2, 0)	10
(0, 0)	0
$\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{100 + 135}{19} = \frac{235}{19}$ ← મહત્તમ

આમ, બિંદુ $\left(\frac{20}{19}, \frac{45}{19}\right)$ આગળ મહત્તમ મૂલ્ય $\frac{235}{19}$ મળે.

21.

➪ ઘટના E_1 : યંત્ર A દ્વારા ઉત્પાદન થયું હોય.

ઘટના E_2 : યંત્ર B દ્વારા ઉત્પાદન થયું હોય.

$P(E_1) = \frac{60}{100}$

$P(E_2) = \frac{40}{100}$

ઘટના A : વસ્તુ પસંદ કરતાં તે ખામીયુક્ત હોય.

$P(A \mid E_1) = 0.02 = \frac{2}{100}$,

$P(A \mid E_2) = 0.01 = \frac{1}{100}$

વસ્તુ પસંદ કરતાં ખામીયુક્ત હોય અને B દ્વારા ઉત્પાદિત થઈ હોય તેની સંભાવના,

$\therefore P(E_2 \mid A) = \frac{P(E_2) \cdot P(A \mid E_2)}{P(A)}$

$\therefore P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)$

$= \frac{60}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{1}{100}$

$= \frac{120}{10000} + \frac{40}{10000}$

$= \frac{160}{10000}$

$\therefore P(E_2 \mid A) = \frac{\frac{40}{100} \times \frac{1}{100}}{\frac{160}{10000}}$

$= \frac{1}{4}$

વિભાગ-C

22.

➪ $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

હવે, $A'A = I$

$\therefore \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} 0+4y^2+z^2 & 0+2y^2-z^2 & 0-2y^2+z^2 \\ 0+2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ 0-2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore 4y^2 + z^2 = 1$ ... (1)

$2y^2 - z^2 = 0$ ... (2)

$x^2 + y^2 + z^2 = 1$ ... (3)

$x^2 - y^2 - z^2 = 0$ ... (4)

સમીકરણ (1) અને (2) ઉકેલતાં,

$4y^2 + z^2 = 1$

$2y^2 - z^2 = 0$

$6y^2 = 1$

$\therefore y^2 = \frac{1}{6}$

$\therefore y = \pm\frac{1}{\sqrt{6}}$

સમીકરણ (2) પરથી, $2 \times \frac{1}{6} - z^2 = 0$ ઉકેલતાં,

$\therefore z^2 = \frac{1}{3}$

$\therefore z = \pm\frac{1}{\sqrt{3}}$

$z^2 = \frac{1}{3}$, $y^2 = \frac{1}{6}$ કિંમત સમીકરણ (3)માં મૂકતાં,

$x^2 + \frac{1}{6} + \frac{1}{3} = 1$

$\therefore x^2 = 1 - \frac{1}{6} - \frac{1}{3}$

$= \frac{6-1-2}{6}$

$= \frac{1}{2}$

$\therefore x = \pm\frac{1}{\sqrt{2}}$

આમ, $x = \pm\frac{1}{\sqrt{2}}$,

$y = \pm\frac{1}{\sqrt{6}}$,

$z = \pm\frac{1}{\sqrt{3}}$

23.

➪ ધારો કે, $\frac{1}{x} = a$, $\frac{1}{y} = b$, $\frac{1}{z} = c$

$2a + 3b + 10c = 4$

$4a - 6b + 5c = 1$

$6a + 9b - 20c = 2$

➪ શ્રેણિક સ્વરૂપે લખતાં,

$\therefore \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\therefore AX = B$

જ્યાં, $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$AX = B$

$\therefore X = A^{-1}B$

➪ A^{-1} શોધવા માટે,

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$
$= 2(75) - 3(-110) + 10(72)$
$= 150 + 330 + 720$
$= 1200 \neq 0$

$\therefore$ A^{-1} નું અસ્તિત્વ છે.

➪ *adj* A મેળવવા માટે,

2નો સહઅવયવ $A_{11} = (-1)^2 \begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix}$
$= 1(120 - 45)$
$= 75$

3નો સહઅવયવ $A_{12} = (-1)^3 \begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix}$
$= (-1)(-80 - 30)$
$= 110$

10નો સહઅવયવ $A_{13} = (-1)^4 \begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix}$
$= 1(36 + 36)$
$= 72$

4નો સહઅવયવ $A_{21} = (-1)^3 \begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix}$
$= (-1)(-60 - 90)$
$= 150$

–6નો સહઅવયવ $A_{22} = (-1)^4 \begin{vmatrix} 2 & 10 \\ 6 & -20 \end{vmatrix}$
$= 1(-40 - 60)$
$= -100$

5નો સહઅવયવ $A_{23} = (-1)^5 \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix}$
$= (-1)(18 - 18)$
$= 0$

6નો સહઅવયવ $A_{31} = (-1)^4 \begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix}$
$= 1(15 + 60)$
$= 75$

9નો સહઅવયવ $A_{32} = (-1)^5 \begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix}$
$= (-1)(10 - 40)$
$= 30$

–20નો સહઅવયવ $A_{33} = (-1)^6 \begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix}$
$= 1(-12 - 12)$
$= -24$

$$adj\ A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ adj\ A$$

$$A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

➪ $X = A^{-1}B$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\therefore \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$

ઉકેલ : $x = 2, y = 3, z = 5$

24.

➪ $(x - a)^2 + (y - b)^2 = c^2$ .... (1)

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$2(x - a) + 2(y - b)\frac{dy}{dx} = 0$$

$$\therefore (x - a) + (y - b)\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{-(x-a)}{y-b} \quad \text{.......... (2)}$$

હવે, બંને બાજુ x પ્રત્યે પુનઃ વિકલન કરતાં,

$$\frac{d^2y}{dx^2} = -\left[\frac{(y-b)(1)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2}\right]$$

$$= -\left[\frac{(y-b)+\frac{(x-a)(x-a)}{(y-b)}}{(y-b)^2}\right] \quad (\because \text{પરિણામ-2})$$

$$= -\left[\frac{(y-b)^2+(x-a)^2}{(y-b)^3}\right]$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-c^2}{(y-b)^3} \quad (\because \text{પરિણામ-1})$$

હવે, $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}} = \dfrac{\left[1+\left(\dfrac{x-a}{y-b}\right)^2\right]^{\frac{3}{2}}}{\dfrac{-c^2}{(y-b)^3}}$

$$= \frac{\left[\dfrac{(y-b)^2+(x-a)^2}{(y-b)^2}\right]^{\frac{3}{2}}}{\dfrac{-c^2}{(y-b)^3}}$$

$$= \frac{-[c^2]^{\frac{3}{2}}}{[(y-b)^2]^{\frac{3}{2}}} \times \frac{(y-b)^3}{[c^2]}$$

$$= \frac{-c^3}{(y-b)^3} \times \frac{(y-b)^3}{c^2}$$

$= -c, \quad c > 0$

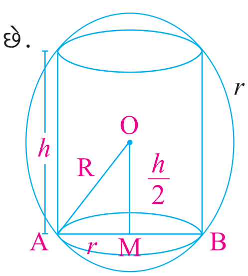
જે a અને b પર આધારિત ન હોય તેવો અચળ છે.

25.

અહીં, ગોળાની ત્રિજ્યા R આપેલ છે.

ધારો કે નળાકારની ત્રિજ્યા r અને ઊંચાઈ h છે.

કાટકોણ ΔOMA પરથી,



$R^2 = r^2 + \dfrac{h^2}{4}$ .... (1)

→ નળાકારનું ઘનફળ (V) $= \pi r^2 h$

$\therefore \quad V = \pi\left(R^2 - \dfrac{h^2}{4}\right)(h)$

$\therefore \quad f(h) = \pi\left(R^2 h - \dfrac{h^3}{4}\right)$

$f'(h) = \pi\left(R^2 - \dfrac{3h^2}{4}\right)$

$f''(h) = \pi\left(0 - \dfrac{6h}{4}\right)$

$f''(h) = \dfrac{-3\pi h}{2} < 0$

→ નળાકારનું મહત્તમ ઘનફળ મેળવવા માટે,

$f'(h) = 0$

$\therefore \pi\left(R^2 - \dfrac{3h^2}{4}\right) = 0$

$\therefore \quad R^2 = \dfrac{3h^2}{4}$

$\therefore \quad R = \dfrac{\sqrt{3}\,h}{2}$

$\therefore \quad h = \dfrac{2R}{\sqrt{3}}$

→ નળાકારનું ઘનફળ, V $= \pi\left(R^2 - \dfrac{h^2}{4}\right)(h)$

$= \pi\left(R^2 - \dfrac{4R^2}{4(3)}\right)\left(\dfrac{2R}{\sqrt{3}}\right)$

$= \pi\left(\dfrac{2R^2}{3}\right)\left(\dfrac{2R}{\sqrt{3}}\right)$

$= \dfrac{4\pi R^3}{3\sqrt{3}}$

26.

$I = \displaystyle\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

$5x + 3 = A\dfrac{d}{dx}(x^2 + 4x + 10) + B$

$5x + 3 = A(2x + 4) + B$

$5x + 3 = 2Ax + 4A + B$

→ x નો સહગુણક તથા અચળ પદને સરખાવતાં,

$\therefore 2A = 5$ | $\therefore 4A + B = 3$

$\therefore A = \dfrac{5}{2}$ | $\therefore 4\left(\dfrac{5}{2}\right) + B = 3$

$\therefore 10 + B = 3$

$\therefore B = -7$

$$= \int \frac{\frac{5}{2}\cdot\frac{d}{dx}(x^2+4x+10) - 7}{\sqrt{x^2+4x+10}}\,dx$$

$$= \frac{5}{2}\int \frac{\frac{d}{dx}(x^2+4x+10)}{\sqrt{x^2+4x+10}}\,dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}\,dx$$

$$= \frac{5}{2}\int (x^2+4x+10)^{-\frac{1}{2}}\,\frac{d}{dx}(x^2+4x+10)\,dx - 7\int \frac{dx}{\sqrt{x^2+2(2x)+4-4+10}}$$

$$= \frac{5}{2}\int (x^2+4x+10)^{-\frac{1}{2}}\,\frac{d}{dx}(x^2+4x+10)\,dx - 7\int \frac{dx}{\sqrt{(x+2)^2+(\sqrt{6})^2}}$$

$$= \frac{5}{2}\int \frac{(x^2+4x+10)^{\frac{1}{2}}}{\frac{1}{2}} - 7\log\left|x + 2 + \sqrt{x^2+4x+10}\right| + c$$

$$\therefore I = 5\sqrt{x^2+4x+10} - 7\log\left|x + 2 + \sqrt{x^2+4x+10}\right| + c$$

27.

$(x^3 + x^2 + x + 1)\dfrac{dy}{dx} = 2x^2 + x$

$\therefore dy = \dfrac{(2x^2 + x)}{(x^3 + x^2 + x + 1)}\, dx$

$\therefore dy = \dfrac{(2x^2 + x)\, dx}{x^2(x+1) + 1(x+1)}$

$\therefore dy = \dfrac{(2x^2 + x)\, dx}{(x^2+1)(x+1)}$

→ બંને બાજુ સંકલન કરતાં,

$\therefore \int 1\, dy = \int \dfrac{(2x^2 + x)\, dx}{(x^2+1)(x+1)}$

હવે, $\dfrac{2x^2 + x}{(x+1)(x^2+1)} = \dfrac{A}{x+1} + \dfrac{Bx + C}{x^2+1}$

$\therefore 2x^2 + x = A(x^2 + 1) + (Bx + C)(x + 1)$

$x = -1$ લેતાં,

$\therefore 2(-1)^2 - 1 = A(2) + (Bx + C)(0)$

$\therefore 2 - 1 = 2A$

$\therefore A = \dfrac{1}{2}$

$x = 0$ લેતાં,

$\therefore 0 = A(1) + B(0) + C(1)$

$\therefore 0 = \dfrac{1}{2} + C$

$\therefore C = -\dfrac{1}{2}$

$x = 1$ લેતાં,

$\therefore 2(1) + 1 = A(2) + (B + C)(2)$

$\therefore 3 = 2A + 2B + 2C$

$\therefore 3 = 2\left(\dfrac{1}{2}\right) + 2B + 2\left(-\dfrac{1}{2}\right)$

$\therefore 3 = 1 + 2B - 1$

$\therefore B = \dfrac{3}{2}$

$\therefore \int 1\, dy = \dfrac{1}{2}\int \dfrac{dx}{x+1} + \int \dfrac{\left(\frac{3}{2}x - \frac{1}{2}\right)}{x^2+1}\, dx$

$\therefore \int 1\, dy = \dfrac{1}{2}\int \dfrac{dx}{x+1} + \dfrac{3}{2}\int \dfrac{x\, dx}{x^2+1} - \dfrac{1}{2}\int \dfrac{dx}{x^2+1}$

$\therefore \int 1\, dy = \dfrac{1}{2}\int \dfrac{dx}{x+1} + \dfrac{3}{4}\int \dfrac{2x\, dx}{x^2+1} - \dfrac{1}{2}\int \dfrac{dx}{x^2+1}$

$\therefore \int 1\, dy = \dfrac{1}{2}\int \dfrac{dx}{x+1} + \dfrac{3}{4}\int \dfrac{\frac{d}{dx}(x^2+1)}{x^2+1}\, dx - \dfrac{1}{2}\int \dfrac{dx}{x^2+1}$

$\therefore y = \dfrac{1}{2}\, log|x + 1| + \dfrac{3}{4}\, log\,|x^2 + 1| - \dfrac{1}{2}\, tan^{-1}x + k$ ... (1)

હવે, $x = 0$ અને $y = 1$ હોય ત્યારે,

$\therefore 1 = \dfrac{1}{2}\, log|1| + \dfrac{3}{4}\, log\,|1| - \dfrac{1}{2}\, tan^{-1}(0) + c$

$\therefore 1 = 0 + 0 + 0 + k$

$\therefore k = 1$

kની કિંમત પરિણામ (1) માં મૂકતાં,

$\therefore y = \dfrac{1}{2}\, log|x + 1| + \dfrac{3}{4}\, log\,|x^2 + 1| - \dfrac{1}{2}\, tan^{-1}x + 1$

$\therefore y = \dfrac{2}{4}\, log|x + 1| + \dfrac{3}{4}\, log\,|x^2 + 1| - \dfrac{1}{2}\, tan^{-1}x + 1$

$\therefore y = \dfrac{1}{4}\, [log(x + 1)^2 + log(x^2 + 1)^3] - \dfrac{1}{2}\, tan^{-1}x + 1$

$\therefore y = \dfrac{1}{4}\, [log[(x + 1)^2(x^2 + 1)^3] - \dfrac{1}{2}\, tan^{-1}x + 1;$

જે આપેલ વિકલ સમીકરણનો વિશિષ્ટ ઉકેલ છે.